

Final Technical Report  
for  
"Nonlinear Dynamics for Communication Systems"  
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Principal Investigator

## 1 Summary of Accomplishments

We have followed closely the tasks and approaches presented in our original research proposal, dated September 20, 1996. During the 3-year grant period, we have not only successfully accomplished all of the proposed tasks, but have discovered and developed several new results of fundamental significance and potential applications on the following areas: 1) robustness of chaotic synchronization schemes, 2) efficiency of chaotic synchronization systems, 3) design of practical chaotic spread-spectrum communication systems, 4) channel capacity of chaotic spread-spectrum communication systems, and 5) potential commercial applications of chaotic spread-spectrum communication systems.

In the area of robustness of chaotic synchronization schemes, we have developed a rigorous theory of  $H_\infty$  robustness for chaotic synchronization of two chaotic systems. This theory is indispensable for designing robust transmitters and receivers for chaotic spread-spectrum communications.

To enhance the competitive edge of chaotic spread-spectrum communication systems over current commercial CDMA systems, the bandwidth for transmitting the chaotic synchronization signals must be reduced significantly. We have made a major breakthrough on this critical issue by inventing a totally new chaotic synchronization scheme called "impulsive chaotic synchronization". Unlike other chaotic synchronization schemes, which are based on ordinary differential equations (or difference equations), our impulsive synchronization approach is based on the recently developed mathematical theory of impulsive differential equations. Although impulsive synchronization appears to be intuitively possible, no theoretical analysis had been made prior our invention. We have published a rigorous theoretical foundation of our invention and have presented experimental confirmation on the impulsive synchronization of two Chua's oscillators. Based on our impulsive synchronization scheme, the bandwidth of the transmitted synchronization signals can be reduced by at least 30,000 times compared to other state-of-the-art **continuous** chaotic synchronization schemes. The channel bandwidth problem which had haunted many previous synchronization schemes has now been successfully solved because our impulsive synchronization signals are transmitted **digitally**. Our chaotic

impulsive synchronization scheme also makes it possible for a complete **digital** implementation of the baseband chaotic components for both transmitters and receivers.

Our research on the above cited chaotic impulsive synchronization scheme has led to the invention of a brand new technology called "Chaotic Digital Code-Division Multiple Access for Wireless Communication Systems". A patent based on this invention has been filed by the University of California, Office of Technology Licensing, under Case No. B-97-080. We have demonstrated by extensive computer simulations that chaotic digital CDMA communications based on our chaotic impulsive synchronization technology can double the channel capacity of CDMA in wireless environments. In addition, chaotic digital CDMA systems can also be used in cable TV networks for Internet access via coaxial cables. Comparing to the current **synchronous** CDMA scheme used in cable TV networks for Internet access, our **non-synchronous** chaotic digital CDMA can provide at least a 1.5 times bigger channel capacity.

The key concept of our chaotic digital CDMA system consists of substituting the **chaotic carrier** generator for both the chip generator and the carrier generator required in current CDMA systems. This significantly simplifies the hardware complexity of both transmitters and receivers, thereby making it possible to design other novel applications, such as home automation via power-line carriers. Because the high-frequency chip sequence has been eliminated, our chaotic digital CDMA system can work in a unique mode, called the **interleave** mode, where the transmitter has to transmit only a small portion of the entire message bit duration. By doing so, the interference level within each cell is reduced significantly. This results in a bigger global channel capacity for chaotic digital communication systems. Because of the relatively low efficiency of the power amplification stage in RF transmitters, the interleave mode will result in a considerable longer battery life than conventional CDMA systems. This advantage is particularly important for mobile stations in wireless environments.

We have also found and confirmed that a "lossy" form of synchronization between chaotic systems, called *generalized chaotic synchronization*, is much more robust to both channel distortion and parameter drifts than conventional chaotic synchronization. Because the generalized chaotic synchronization schemes play a critical role in the chaotic secure systems, our findings that two chaotic systems can be generally synchronized via infinitely many linear synchronizing manifolds represent a major breakthrough in the theoretical research of this field. Before our work, other researchers could only use computer simulations to visualize the unknown synchronizing manifolds and could not find closed forms. This difficulty presented some major obstacles to the applications of generalized chaotic synchronization to secure communication systems. Among many are: determining 1) how many synchronizing manifolds a certain system can have, 2) what is the distance between two different synchronizing manifolds, and 3) how secure a certain generalized chaotic synchronization is. Our theoretical

findings provide a definitive answer to the above questions.

In view of the advantages of chaotic impulsive synchronization over chaotic continuous synchronization, some recent progress has been made to improve many different aspects of chaotic communication systems based on impulsive synchronization.

The first aspect is the theoretical study of impulsive control theory under more general and practical conditions. In the physical world, it is impossible to get the exact parameters and noise model. To guarantee that the theoretical results can predict the performance of the whole system under realistic conditions, we need to study the stability of impulsive synchronization in a certain range, and not simply around a single equilibrium point. This kind of stability is called *practical stability*. We have successfully developed a theory to give the error range of impulsive control algorithms under practical stability criteria. We have also developed the theoretical basis for practical stability of impulsive synchronizations.

The second aspect is to improve the implementability of chaotic digital CDMA((CD)<sup>2</sup>MA) systems by evaluating their performance under different channel conditions and different system configurations. One of the main problems is the full digitization of the entire (CD)<sup>2</sup>MA system, including the chaotic core. This conversion calls into question the stability of the entire system due to the sampling process of the digital controller. We have made significant advances in developing a theoretical framework for the stability of chaotic systems under sampled-data control. We give a theory to guarantee the asymptotic stability of a chaotic system controlled by sampled-data feedback. This theory can be easily used to study the digitized chaotic synchronization schemes.

We had previously implemented a hardware test-bed for studying impulse synchronization experimentally. Based on this hardware experimental platform we have discovered many exciting results. First, we experimentally verified that impulsive synchronization is much more robust than its continuous counterparts to noise and parameter drifts. Second, we have observed the successful impulse synchronization between two *hyperchaotic* systems through a single synchronizing channel. Third, different chaotic secure communication schemes and spread-spectrum communication schemes were successfully implemented in this single test-bed. Since hyperchaotic systems can provide much more security in a chaotic communication system, our experimental results on the impulsive chaotic communication systems based on hyperchaotic systems are very important for future research.

From the experimental point of view, it is not so easy to synchronize two actual chaotic circuits by using impulsive synchronization in view of the following reasons:

- In actual circuits, noise is unavoidable.
- Parameter mismatch and changes (drifts) in the component parameters of the chaotic circuits in the transmitter and the receiver are unavoidable.

- The width of the impulse  $Q$  can not be chosen too small, especially when the circuits are hyperchaotic, because noise and parameter mismatch will soon desynchronize the chaotic circuits even though the two circuits have synchronized during the period when synchronization impulses are present.

We have successfully designed and carried out experimental results on impulsive synchronization of two kinds of chaotic circuits; namely, Chua's oscillator and a hyperchaotic circuit, are presented. To impulsively synchronize two Chua's oscillators, synchronization impulses sampled from *one* state variable of the driving circuit are transmitted to the driven circuit. To impulsively synchronize two hyperchaotic circuits, synchronizing impulses sampled from *two* signals of the driving circuit are sent to the driven circuit. Our experimental results show that the accuracy of impulsive synchronization depends on both the period and the width of the impulse. The ratio between the impulse width and impulse period for "almost-identical" *synchronization* increases as the impulse period increases. The robustness of impulsive synchronization to additive noise was also experimentally studied. For sufficiently short impulse periods, no significant differences are observed between impulsive and continuous synchronizations. The performance of chaotic spread spectrum communication systems based on impulsive synchronization was also studied experimentally.

## 2 List of Publications:

In the following, we present a chronological list of all publications that have resulted from research supported, either partially or in full, by the above cited ONR grant.

1. Suykens, J.A.K., Curran, P.F., Vandewalle, J. and Chua, L.O. "Robust nonlinear  $H_\infty$  synchronization of chaotic Lur'e systems." *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, Oct. 1997, vol.44, (no.10):891-904.
2. Kolumban, G., Kennedy, M.P. and Chua, L.O. "The role of synchronization in digital communications using chaos. I. Fundamentals of digital communications." *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, Oct. 1997, vol.44, (no.10):927-36.
3. Tao Yang and Chua, L.O. "Impulsive stabilization for control and synchronization of chaotic systems: theory and application to secure communication." *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, Oct. 1997, vol.44, (no.10):976-88.
4. Suykens, J.A.K., Curran, P.F., Tao Yang, Vandewalle, J. and Chua, L.O. "Nonlinear  $H_\infty$  synchronization of Lur'e systems: dynamic output feedback case." *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, Nov. 1997, vol.44, (no.11):1089-92.
5. Guo-Qun Zhong, Chai Wah Wu and Chua, L.O. "Torus-doubling bifurcations in four mutually coupled Chua's circuits." *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, Feb. 1998, vol.45, (no.2):186-93.
6. Gang Hu, Jinghua Xiao, Chua, L.O. and Pivka, L. "Controlling spiral waves in a model of two-dimensional arrays of Chua's circuits." *Physical Review Letters*, 2 March 1998, vol.80, (no.9):1884-7.
7. A.I. Panas, T. Yang and Leon O. Chua, "Experimental Results of Impulsive Synchronization Between Two Chua's Circuits", *International Journal of Bifurcation and Chaos*, vol.8, (no.3), March 1998.
8. J.A.K. Suykens, T. Yang and Leon O. Chua, "Impulsive synchronization of chaotic Lur'e systems by measurement feedback," *International Journal of Bifurcation and Chaos*, vol.8, (no.6), June 1998.

9. T. Yang, J.A.K. Suykens and Leon O. Chua, "Impulsive Control of Nonautonomous Chaotic Systems Using Practical Stabilization," *International Journal of Bifurcation and Chaos*, vol.8, (no.7), July 1998.
10. T. Yang and Leon O. Chua, "Applications of Chaotic Digital Code-Division Multiple Access(CDMA) to Cable Communication Systems," *International Journal of Bifurcation and Chaos*, vol.8, (no.8), Aug. 1998.
11. T. Yang and Leon O. Chua, "Error Performance for Chaotic Digital Code-Division Multiple Access(CDMA)," *International Journal of Bifurcation and Chaos*, vol.8, (no.10), Oct. 1998.
12. Kolumban, G.; Kennedy, M.P.; Chua, L.O. The role of synchronization in digital communications using chaos. II. Chaotic modulation and chaotic synchronization. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, Nov. 1998, vol.45, (no.11):1129-40.
13. T. Yang; Chua, L.O. Control of chaos using sampled-data feedback control. *International Journal of Bifurcation and Chaos in Applied Sciences and Engineering*, Dec. 1998, vol.8, (no.12):2433-8.
14. Chua, L.O.; Wu, C.W.; Zhong, G.-O.; Liu, L.F. Synthesizing arbitrary driving-point and transfer characteristics. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, Dec. 1998, vol.45, (no.12):1225-32.
15. T. Yang; Chua, L.O. Generalized synchronization of chaos via linear transformations. *International Journal of Bifurcation and Chaos in Applied Sciences and Engineering*, Jan. 1999, vol.9, (no.1):215-219.
16. Moiola, J.L.; Chua, L.O. Hopf bifurcations and degeneracies in Chua's circuit-a perspective from a frequency domain approach. *International Journal of Bifurcation and Chaos in Applied Sciences and Engineering*, Jan. 1999, vol.9, (no.1):295-303.
17. T. Yang, "Impulsive control, " *IEEE Transactions on Automatic Control*, May 1999, vol.44, (no.5):1081-1083.
18. M. Itoh, T. Yang, and L. O. Chua, Experimental Study of Impulsive Synchronization of Chaotic and Hyperchaotic Circuits, *International Journal of Bifurcation and Chaos*, vol.9, (no.7), July 1999.

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